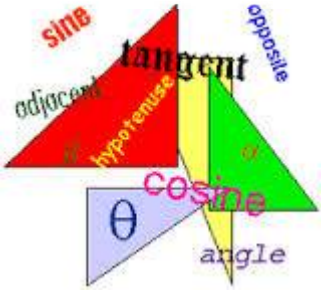
Chapter – 13

Trigonometry



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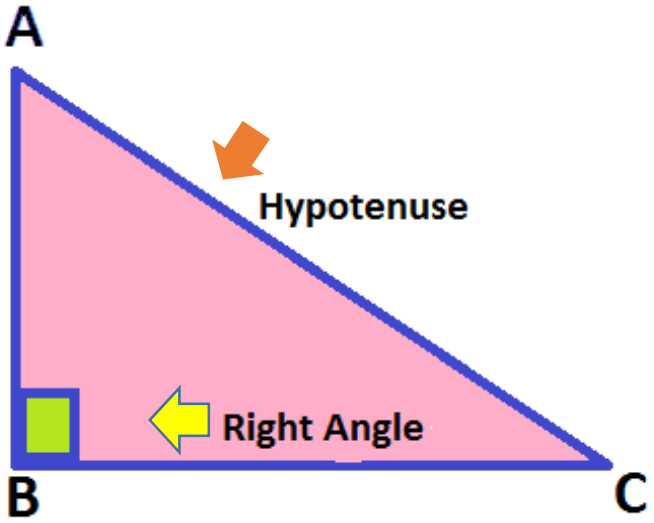
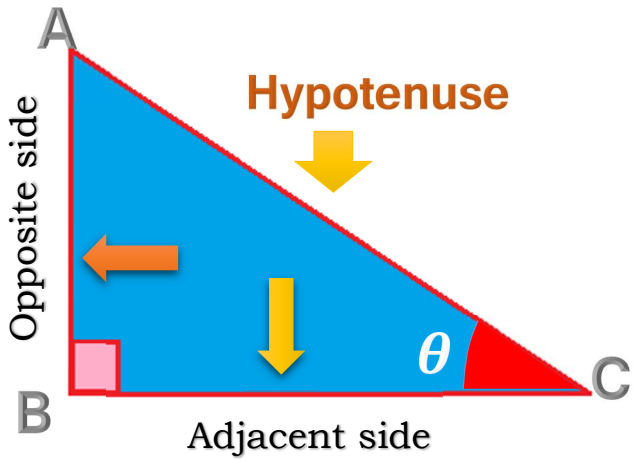
SSLC CLASS NOTES CHAPTER-13 : TRIGONOMETRY

Trigonometry:

- Related to Right angled triangle



- Suppose we mark the angles other than the right angle. We can mark either ∠BAC or ∠ACB, which are always acute angles. The marked angle is denoted as 'θ' (Greek letter - read as theta)
- The side which is opposite to θ is called opposite side and the other one is adjacent.
- The sides related to "θ"



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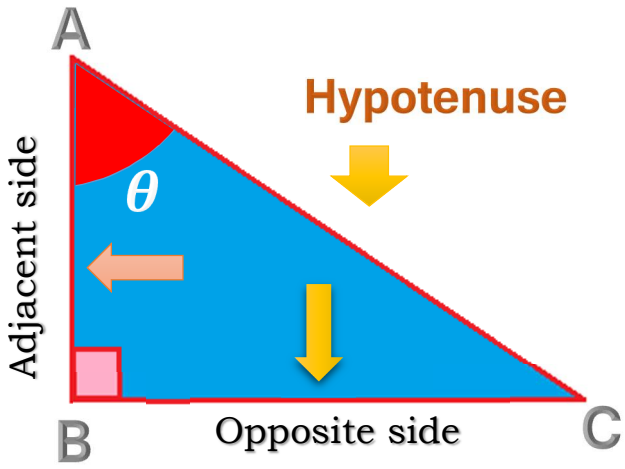
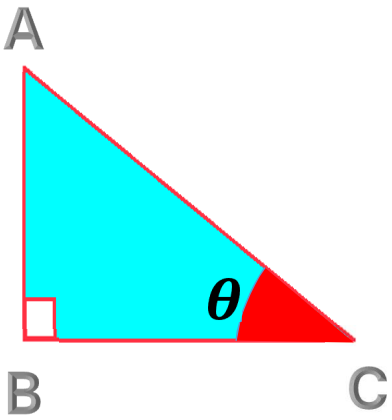




$\sin\theta$	$\frac{\text{Opposite side}}{\text{Hypotenuse}}$	$\frac{AB}{AC}$	$\frac{1}{\text{Cosec}\theta}$
$\text{Cos}\theta$	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\frac{BC}{AC}$	$\frac{1}{\sec\theta}$
$\text{Tan}\theta$	$\frac{\text{Opposite side}}{\text{Adjacent side}}$	$\frac{AB}{BC}$	$\frac{1}{\cot\theta}$
$\text{Cosec}\theta$	$\frac{\text{Hypotenuse}}{\text{Opposite side}}$	$\frac{AC}{AB}$	$\frac{1}{\sin\theta}$
$\sec\theta$	$\frac{\text{Hypotenuse}}{\text{Adjacent side}}$	$\frac{AC}{BC}$	$\frac{1}{\cos\theta}$
$\text{Cot}\theta$	$\frac{\text{Adjacent side}}{\text{Opposite side}}$	$\frac{BC}{AB}$	$\frac{1}{\tan\theta}$

Formulae
1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \cot^2\theta = \text{cosec}^2\theta$
3. $\tan^2\theta + 1 = \sec^2\theta$

SSLC CLASS NOTES CHAPTER-13 : TRIGONOMETRY

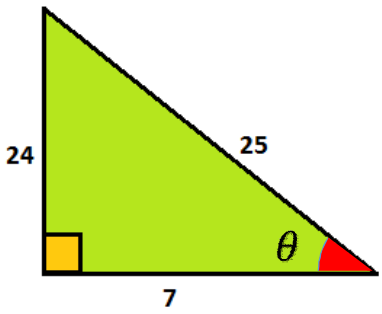
	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
$\csc\theta$	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
$\cot\theta$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometry ratios of complimentary angles			
sin(	cos θ	cosec(	sec θ
cos(	sin θ	sec(	cosec θ
tan(	cot θ	cot(	tan θ

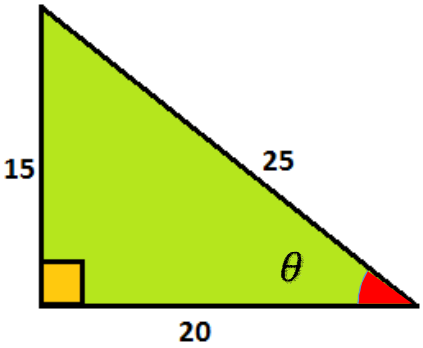
SSLC CLASS NOTES CHAPTER-13 : TRIGONOMETRY

EXERCISE13.1

I. Find sin θ and cosθ for the following

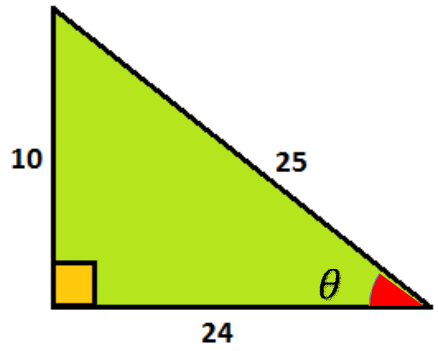


$\sin\theta$	$\frac{\text{Opposidte side}}{\text{Hypotenuse}}$	$\frac{24}{25}$
$\cos\theta$	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\frac{7}{25}$



$\sin\theta$	$\frac{\text{Opposidte side}}{\text{Hypotenuse}}$	$\frac{15}{25}$
$\cos\theta$	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\frac{20}{25}$

SSLC CLASS NOTES CHAPTER-13 : TRIGONOMETRY



sinθ	**$\frac{\text{Opposidte side}}{\text{Hypotenuse}}$**	**$\frac{10}{25}$**
Cosθ	**$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$**	**$\frac{24}{25}$**

II. Find the following :

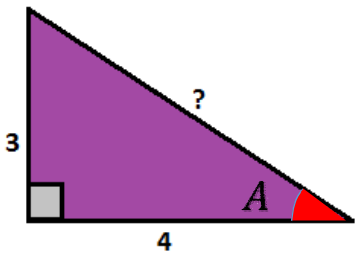
1. If sin x = $\frac{3}{5}$, cosec x = $\frac{5}{3}$
2. If cos x = $\frac{24}{25}$, sec x = $\frac{25}{24}$
3. If tan x - $\frac{7}{24}$, cot x = $\frac{24}{7}$
4. If cosec x = $\frac{25}{15}$, sin x = $\frac{15}{25}$
5. If sin A = $\frac{3}{5}$ and cos A = $\frac{4}{5}$, tan A =$\frac{3}{4}$
6. If cot A = $\frac{8}{15}$ and sin A = $\frac{15}{17}$, cos A = $\frac{8}{17}$

III. Solve :

1. Given tan A = $\frac{3}{5}$,find the value of sin A and cos A..

   tan A= $\frac{\text{Hypotenuse}}{\text{Adjacent side}}$ = $\frac{3}{4}$

   By Pythagoras Theorem
   $(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$
   $(\text{Hypotenuse})^2 = 3^2 + 4^2$



$(\text{Hypotenuse})^2 = 9 + 16$

$(\text{Hypotenuse})^2 = 25$

$\text{Hypotenuse} = \sqrt{25}$

$\text{Hypotenuse} = 5$

$\text{Sin A} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

$\text{Sin A} = \frac{3}{5}$

$\text{Cos A} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

$\text{Cos A} = \frac{4}{5}$

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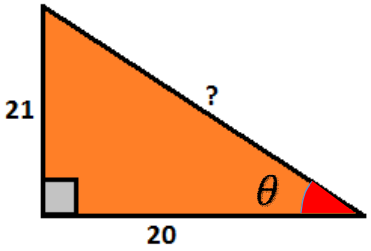
2. Given $\cot\theta = \frac{20}{21}$ determine cos □ and cosec □□

$\text{Cot}\ \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{20}{21}$

By Pythagoras Theorem

$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

$(\text{Hypotenuse})^2 = 21^2 + 20^2$



$(\text{Hypotenuse})^2 = 441 + 400$

$(\text{Hypotenuse})^2 = 841$

$\text{Hypotenuse} = \sqrt{841}$

$\text{Hypotenuse} = 29$

$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

$\cos\theta = \frac{20}{29}$

$\text{cosec}\ \theta = \frac{\text{Hypotenuse}}{\text{Opposite side}}$

$\text{cosec}\ \theta = \frac{29}{21}$

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3. Given $\tan A = \frac{7}{24}$,Find the other trigonometric ratio of angle A.

$\tan A = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{7}{24}$

By Pythagoras Theorem

$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

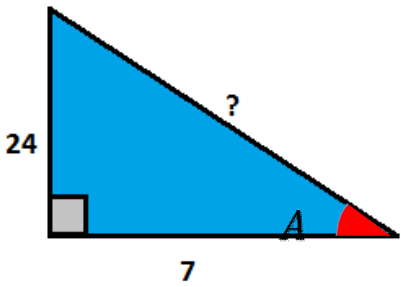
$(\text{Hypotenuse})^2 = 7^2 + 24^2$

$(\text{Hypotenuse})^2 = 49 + 576$

$(\text{Hypotenuse})^2 = 625$

$\text{Hypotenuse} = \sqrt{625}$

$\text{Hypotenuse} = 25$



$\text{Sin A} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{7}{25}$

$\text{Cos A} = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{24}{25}$

$\text{cosec A} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{25}{7}$

$\text{sec A} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{25}{24}$

$\text{Cot A} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{24}{7}$

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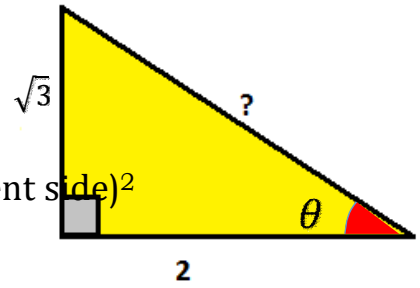
4. If $2 \sin\theta = \sqrt{3}$, Find $\cos\theta$, $\tan\theta$ and $\cot\theta + \text{cosec}\,\theta$.

$2 \sin\theta = \sqrt{3}$



$\sin\theta = \frac{\sqrt{3}}{2} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

$2^2 = (\sqrt{3})^2 + (\text{Adjacent side})^2$

$4 = 3 + (\text{Adjacent side})^2$

$4-3 = (\text{Adjacent side})^2$

$1 = (\text{Adjacent side})^2$

$\text{Adjacent side} = 1$

$\text{Cos}\,\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{1}{2}$

$\text{Tan}\,\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{\sqrt{3}}{1}$

$\text{Cot}\,\theta + \text{Cosec}\,\theta$

$= \frac{\text{Adjacent side}}{\text{Opposite side}} + \frac{\text{Hypotenuse}}{\text{Opposite side}}$

$= \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}$

$= \frac{3}{\sqrt{3}}$

$= \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}}$

$= \sqrt{3}$

5. If 3 tan θ = 1, Find sin θ, cos θ and cot θ.

3 tan θ = 1

$\text{Tan } \theta = \frac{1}{3} = \frac{\text{Opposite side}}{\text{Adjacent side}}$

By Pythagoras theorem

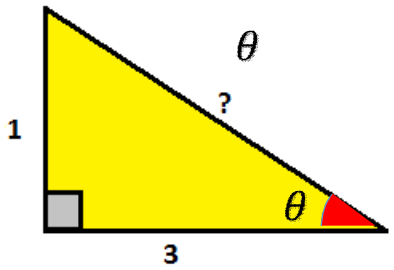
$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

$(\text{Hypotenuse})^2 = 1^2 + 3^2$

$(\text{Hypotenuse})^2 = 1 + 9$

$(\text{Hypotenuse})^2 = 10$

$\text{Hypotenuse} = \sqrt{10}$



$\text{Sin } \theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{1}{\sqrt{10}}$

$\text{Cos } \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{3}{\sqrt{10}}$

$\text{Cot } \theta = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{3}{1}$

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6. If sec x = 2 ,then find sin x, tan x, cot x ಮತ್ತು cot x + cosec x .

$\sec x = 2 = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

$2^2 = (\text{Opposite side})^2 + 1^2$

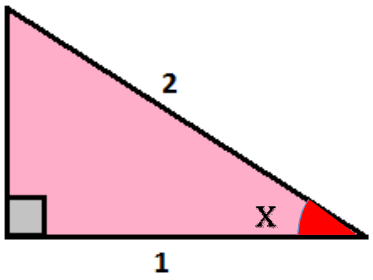
$4 = (\text{Opposite side})^2 + 1$

$4-1 = (\text{Opposite side})^2$

$3 = (\text{Opposite side})^2$

$(\text{Opposite side})^2 = 3$

$\text{Opposite side} = \sqrt{3}$



$\text{Sin } x = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}$

$\text{Cot } x = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{1}{\sqrt{3}}$

$\text{Tan } \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{\sqrt{3}}{1}$

cot x + cosec x

$= \frac{\text{Adjacent side}}{\text{Opposite side}} + \frac{\text{Hypotenuse}}{\text{Opposite side}}$

$= \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}$

$= \frac{3}{\sqrt{3}}$

$= \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}}$

$= \sqrt{3}$

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7. If 4 sin A – 3 cos A = 0 , find sin A, cos A, sec A ಮತ್ತು cosec A.

4 sin A = 3 cos A

$\sin A = \frac{3}{4} \cos A$

$\frac{\sin A}{\cos A} = \frac{3}{4}$

$\text{Tan A} = \frac{3}{4} = \frac{\text{Opposite side}}{\text{Adjacent side}}$

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

$(\text{Hypotenuse})^2 = 3^2 + 4^2$

$(\text{Hypotenuse})^2 = 9 + 16$

$(\text{Hypotenuse})^2 = 25$

$\text{Hypotenuse} = \sqrt{25}$

Hypotenuse = 5

$\text{Sin A} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{3}{5}$

$\text{Cos A} = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{4}{5}$

$\text{cosec A} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{5}{3}$

$\sec A = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{5}{4}$

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8. If 13 sin A = 5 and A is acute,find the value of $\frac{5 \sin A - 2 \cos A}{\tan A}$

13 sin A = 5

$\sin A = \frac{5}{13} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

By Pythagoras theorem

$(\text{Adjacent side})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

$13^2 = 5^2 + (\text{Adjacent side})^2$

$169 = 25 + (\text{Adjacent side})^2$

$169\text{-}25 = (\text{Adjacent side})^2$

$144 = (\text{Adjacent side})^2$

$(\text{Adjacent side})^2 = 144$

Adjacent side $= \sqrt{144}$

Adjacent side $= 12$

$$\frac{5\sin A - 2\cos A}{\tan A}$$

$$= \frac{5\frac{\text{Opposite side}}{\text{Hypotenuse}} - 2\frac{\text{Adjacent side}}{\text{Hypotenuse}}}{\frac{\text{Opposite side}}{\text{Hypotenuse}}}$$

$$= \frac{5\times\frac{5}{13} - 2\times\frac{12}{13}}{\frac{5}{12}}$$

$$= \frac{\frac{25}{13} - \frac{24}{13}}{\frac{5}{12}}$$

$$= \frac{25-24}{13} \text{ x } \frac{5}{12}$$

$$= \frac{1}{13} \text{ x } \frac{5}{12}$$

$$= \frac{12}{65}$$

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9. If $\cos\theta = \frac{5}{13}$ and θ is acute, find the value of $\frac{5\tan\theta + 12\cot\theta}{5\tan\theta - 12\cot\theta}$

$\cos\theta = \frac{5}{13} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

$13^2 = (\text{Opposite side})^2 + 5^2$

$169 = (\text{Opposite side})^2 + 25$

$169 - 25 = (\text{Opposite side})^2$

$144 = (\text{Opposite side})^2$

$(\text{Opposite side})^2 = 144$

Opposite side $= \sqrt{144}$

Opposite side $= 12$

$$\frac{5\tan\theta + 12\cot\theta}{5\tan\theta - 12\cot\theta}$$

$$= \frac{5\times\frac{12}{5} + 12\times\frac{5}{12}}{5\times\frac{12}{5} - 12\times\frac{5}{12}}$$

$$= \frac{12+5}{12-5}$$

$$= \frac{17}{7}$$

10. If $13 \cos\theta - 5 = 0$,find $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

$13 \cos\theta - 5 = 0$

$13 \cos\theta = 5$

$\cos\theta = \frac{5}{13} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Opposite side})^2 + (\text{Adjacent side})^2$

$13^2 = (\text{Opposite side})^2 + 5^2$

$169 = (\text{Opposite side})^2 + 25$

$169-25 = (\text{Opposite side})^2$

$144 = (\text{Opposite side})^2$

$(\text{Opposite side})^2 = 144$

$\text{Opposite side} = \sqrt{144}$

$\text{Opposite side} = 12$

$$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\frac{\text{Opposite side}}{\text{Hypotenuse}} + \frac{\text{Adjacent side}}{\text{Hypotenuse}}}{\frac{\text{Opposite side}}{\text{Hypotenuse}} - \frac{\text{Adjacent side}}{\text{Hypotenuse}}}$$

$$= \frac{\frac{12}{13} + \frac{5}{13}}{\frac{12}{13} - \frac{5}{13}}$$

$$= \frac{\frac{12+5}{13}}{\frac{12-5}{13}}$$

$$= \frac{\frac{17}{13}}{\frac{7}{13}}$$

$$= \frac{17}{7}$$

EXERCISE 13.2

I. Answer the following questions:

(1) What trigonometric ratios of angles from 0^0 to 90^0 are equal to 0? (2) Which trigonometric ratios of angles from 0^0 to 90^0 are equal to 1? (3) Which trigonometric ratios of angles from 0^0 to 90^0 are equal to0.5? (4) Which trigonometric ratios of angles from 0^0 to 90^0 are not defined? (5) Which trigonometric ratios of angles from 0^0 to 90^0 are equal?

(1)What trigonometric ratios of angles from $0^0$to $90^0$are equal to 0?

Sin $0^0 = 0$

Cos $90^0 = 0$

Tan $0^0 = 0$

Cot $90^0 = 0$

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(2) Which trigonometric ratios of angles from 0^0 to 90^0 are equal to 1?

Sin $90^0 = 1$

Cos $0^0 = 1$

Tan $45^0 = 1$

Cosec $90^0 = 1$

Sec $0^0 = 1$

Cot $45^0 = 1$

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(3) Which trigonometric ratios of angles from $0^0$to $90^0$are equal to 0.5 ?

Sin $30^0 = \frac{1}{2}$

Cos $60^0 = \frac{1}{2}$

(4) Which trigonometric ratios of angles from $0^0$to $90^0$are not defined?

Tan 90^0 = ND

Cosec 0^0 = ND

Sec 90^0 = ND

Cot 0^0 = ND

(5) Which trigonometric ratios of angles from 0^0 to 90^0 are equal?

0 = sin 0^0 = cos 90^0 = tan 0^0 = cot 90^0

1/2 = sin 30^0 = cos 60^0

$\sqrt{2}$ = cosec 45^0 = sec 45^0

$\frac{1}{\sqrt{2}}$ = sin 45^0 = cos 45^0

$\frac{\sqrt{3}}{2}$ = sin 60^0 = cos 30^0

1 = sin 90^0 = cos 0^0 = tan 45^0 = cosec 90^0 = sec 0^0 = cot 45^0

2 = cosec 30^0 = sec 60^0

$\frac{2}{\sqrt{3}}$ = cosec 30^0 = sec 60^0

$\frac{1}{\sqrt{3}}$ = tan 30^0 = cot 60^0

$\sqrt{3}$ = tan 60^0 = cot 30^0

ND = tan 90^0 = cosec 0^0 = sec 90^0 = cot 0^0

II. Find the values.($0 \leq \theta \leq 90^0$)

1. $\sqrt{2}\cos\theta = 1$

   $\cos\theta = \frac{1}{\sqrt{2}}$

   $\cos 45^0 = \frac{1}{\sqrt{2}}$

   $\theta = 45^0$

2. $\sqrt{3}\tan\theta = 1$

   $\tan\theta = \frac{1}{\sqrt{3}}$

   $\tan 30^0 = \frac{1}{\sqrt{3}}$

   $\theta = 30^0$

3. $2\sin\theta = \sqrt{3}$

   $\sin\theta = \frac{\sqrt{3}}{2}$

   $\sin 60^0 = \frac{\sqrt{3}}{2}$

$\theta = 60^0$

4. $5 \sin \theta = 0$

$\text{Sin } \theta = 0/5$

$\text{Sin } \theta = 0$

$\text{Sin } 0^0 = 0$

$\theta = 0^0$

5. $3 \tan \theta = \sqrt{3}$

$\text{Tan } \theta = \frac{\sqrt{3}}{3}$

$\text{Tan } \theta = \frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$\text{Tan } \theta = \frac{1}{\sqrt{3}}$

$\text{Tan } 30^0 = \frac{1}{\sqrt{3}}$

$\boldsymbol{\theta = 30^0}$

III. Find the value of the following:

(i). $\sin 30^0 \cos 60^0 - \tan^2 45^0$

$= \frac{1}{2} \text{ x } \frac{1}{2} - 1^2$

$= \frac{1}{4} - 1$

$\boldsymbol{= \frac{1-4}{4}}$

$= -\frac{3}{4}$

ii. $\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$

$= \frac{\sqrt{3}}{2} \text{ x } \frac{\sqrt{3}}{2} + \frac{1}{2} \text{ x } \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{3+1}{4}$

$\boldsymbol{= 1}$

iii . $\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$

$= \frac{1}{2} \text{ x } \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \text{ x} \frac{1}{2}$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$\boldsymbol{= 0}$

iv. $2 \sin^2 30^0 - 3 \cos^2 30^0 + \tan 60^0 + 3 \sin^2 90^0$

$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{\sqrt{3}}{2}\right)^2 + \sqrt{3} + 3(1)^2$

$= 2x\frac{1}{4} - 3x\frac{3}{4} + \sqrt{3} + 3$

$= \frac{1}{2} - \frac{9}{4} + \sqrt{3} + \frac{12}{4}$

$= \frac{2-9+12}{4} + \sqrt{3}$

$= \mathbf{\frac{5}{4} + \sqrt{3}}$

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v. $4 \sin^2 60^0 + 3 \tan^2 30^0 - 8 \sin 45^0 \cos 45^0$

$= 4x\left(\frac{\sqrt{3}}{2}\right)^2 + 3x\left(\frac{1}{\sqrt{3}}\right)^2 - 8x\frac{1}{\sqrt{2}}x\frac{1}{\sqrt{2}}$

$= 4x\frac{3}{4} + 3x\frac{1}{3} - 8x\frac{1}{2}$

$= 3 + 1 - 4$

$= 0$

---

vi. $\frac{Cos\,45^0}{sec30^0 + \text{cosec}\,30^0}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$= \frac{\sqrt{3}}{(2+2\sqrt{3})\sqrt{2}}$

$= \mathbf{\frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})}}$

---

vii. $\frac{4sin^2\,60^0 - cos^2\,45^0}{tan^2\,30^0 + sin^2\,0^0}$

$= \frac{4x\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{1}{\sqrt{3}}\right)^2 + 0}$

$= \frac{3 - \frac{1}{2}}{\frac{1}{3}}$

$= \frac{\frac{5}{2}}{\frac{1}{3}}$

$= \mathbf{\frac{5}{2}x\frac{3}{1}}$

$$= \frac{15}{2}$$

viii. $\frac{sin\ 30^0 + tan\ 45^0 - Cosec60^0}{sec30^0 + cos60^0 + cot45^0}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}}$$

$$= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}}$$

$$= \mathbf{\frac{3\sqrt{3}-4}{3\sqrt{3}+4}}$$

ix. $\frac{5cos^2\ 60^0 + 4\ sec^2\ 30^0 - tan^2\ 45^0}{sin^2\ 30^0 + cos^2\ 30^0}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5x\ \frac{1}{4} + 4x\ \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{64}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{1}$$

$$= \mathbf{\frac{67}{12}}$$

x. $\frac{5sin^2\ 30^0 + cos^2\ 45^0 - 4tan^2\ 30^0}{2sin30^0 + cos\ 30^0 + tan45^0}$

$$= \frac{5x\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 4\left(\frac{1}{\sqrt{3}}\right)^2}{2x\frac{1}{2} + \frac{\sqrt{3}}{2} + 1}$$

$$= \frac{\frac{5}{4} + \frac{1}{2} - \frac{4}{3}}{1 + \frac{1}{2} + 1}$$

$$= \frac{\frac{15+6-16}{12}}{2 + \frac{\sqrt{3}}{2}}$$

$= \frac{\frac{5}{12}}{2+\frac{\sqrt{3}}{2}}$

$= \frac{5}{12}\left(\frac{2}{4+\sqrt{3}}\right)$

$= \frac{5}{6}\left(\frac{1}{4+\sqrt{3}}\right)$

EXERCISE 13.3

I. Show that

1. $(1-\sin^2\theta)\sec^2\theta = 1$

LHS $= \cos^2\theta \times \sec^2\theta$ $[\because 1-\sin^2\theta = \cos^2\theta]$

$\Rightarrow \cos^2\theta \times \frac{1}{\cos^2\theta}$ $[\because \sec^2\theta = \frac{1}{\cos^2\theta}]$

= 1 RHS

2. $(1+\tan^2\theta)\cos^2\theta = 1$

LHS $= \sec^2\theta \times \cos^2\theta$ $[\because 1+\tan^2\theta = \sec^2\theta]$

$\Rightarrow \frac{1}{\cos^2\theta} \times \cos^2\theta$ $[\because \sec^2\theta = \frac{1}{\cos^2\theta}]$

= 1RHS

3. $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) = 1$

LHS $=(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$

$= \sec^2\theta\,(1-\sin^2\theta)$ $[\because 1+\tan^2\theta = \sec^2\theta]$

$= \sec^2\theta \times \cos^2\theta$ $[\because 1-\sin^2\theta = \cos^2\theta]$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta$ $[\because \sec^2\theta = \frac{1}{\cos^2\theta}]$

= 1RHS

4. $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\,cosec\,\theta$

LHS $= \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} + \frac{(1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^2\theta + 1 + \cos\theta^2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1 + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= \frac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2(1 + \cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{2}{\sin\theta}$

$= 2cosec\theta$ RHS

---

5. $\frac{1+\sin\theta}{1-\sin\theta} = (\sec\theta + \tan\theta)^2$

LHS = $\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$

$= \frac{(1+\sin\theta)^2}{1-sin^2\theta}$

$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$

$= \left[\frac{1+\sin\theta}{\cos\theta}\right]^2$

$= \left[\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right]^2$

$= [sec\theta + tan\theta]^2$ RHS

---

6. $\frac{\cos A}{1-tanA} + \frac{\sin A}{1-cotA} = \sin A + \cos A$

LHS = $\frac{\cos A}{1-tanA} + \frac{\sin A}{1-cotA}$

$= \frac{\cos A}{1-\frac{sinA}{cosA}} + \frac{\sin A}{1-\frac{cosA}{sinA}}$

$= \frac{\cos A}{\frac{cosA-sinA}{cosA}} + \frac{\sin A}{\frac{sinA-cosA}{sinA}}$

$= \frac{\cos A cosA}{cosA-sinA} + \frac{\sin A sinA}{sinA-cosA}$

$= \frac{\cos A cosA}{cosA-sinA} - \frac{\sin A sinA}{cosA-sinA}$ [$\because a - b = -(b - a)$]

$= \frac{\cos^2 A - \sin^2 A}{cosA - sinA}$

$= \frac{(\cos A + \sin A)(\cosA - \sinA)}{cosA - sinA}$

= (sin A + cos A) RHS

---

7. $\frac{1-tan^2 A}{1+tan^2 A}$ = 1- 2 $\sin^2$ A

LHS = $\frac{1-tan^2 A}{1+tan^2 A}$

$= \frac{1-\frac{sin^2 A}{cos^2 A}}{1+\frac{sin^2 A}{cos^2 A}}$

$= \frac{\frac{cos^2 A - sin^2 A}{cos^2 A}}{\frac{cos^2 A + sin^2 A}{cos^2 A}}$

$= \frac{cos^2 A - sin^2 A}{cos^2 A + sin^2 A}$

$= \frac{1 - sin^2 A - sin^2 A}{1}$

= $1 - 2sin^2 A$ RHS

---

8. $(\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cos \theta$

LHS = $(\sin \theta + \cos \theta)^2$

= $sin^2\theta + \cos^2\theta + 2 \sin \theta \cos \theta$

= 1 + 2 sin θ cos θ RHS

9. Sin A cos A tan A + cos A sin A cot A = 1

LHS = Sin A cos A tan A + cos A sin A cot A

= Sin A cos A $\frac{sin A}{CosA}$ + cos A sin A $\frac{cos A}{sinA}$

= Sin A sin A + cos A cos A

= sin^2 A + $\cos^2$A

= 1 RHS

10. $\frac{\tan A-\sin A}{sin^2 A} = \frac{\tan A}{1+\cos A}$

LHS $= \frac{\tan A-\sin A}{sin^2 A}$

$= \frac{\frac{\text{Sin}A}{CosA}-\sin A}{1-cos^2 A}$

$= \frac{\frac{\text{Sin}A-\text{cos}A\text{sin}A}{CosA}}{(1+\cos A)(1-\cos A)}$

$= \frac{\frac{\text{Sin}A(1-\text{cos}A)}{CosA}}{(1+\cos A)(1-\cos A)}$

$= \frac{\text{Sin}A}{CosA(1+\cos A)}$

---

11. $\text{Tan}^2\ A - \sin^2 A = \tan^2 A \sin^2 A$

LHS $= \text{Tan}^2\ A - \sin^2 A$

$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A$

$= \sin^2 A \left(\frac{1}{\cos^2 A} - 1\right)$

$= \sin^2 A\ (\sec^2 A - 1)$

$= \tan^2 A \sin^2 A$ RHS

---

12. $\text{Cos}^2\ A - \sin^2 A = 2\cos^2 A - 1$

LHS $= \text{Cos}^2\ A - \sin^2 A$

$= \text{Cos}^2\ A - (1 - \cos^2 A)$

$= \text{Cos}^2\ A - 1 + \cos^2 A$

$= 2\text{Cos}^2\ A - 1$ RHS

SSLC CLASS NOTES CHAPTER-13 : TRIGONOMETRY

EXERCISE 13.4

1. Evaluate:

i. $\frac{\tan 65^0}{cot\ 25^0}$

$= \frac{\tan(90-25)^0}{cot\ 25^0}$

$= \frac{cot\ 25^0}{cot\ 25^0}$

$= 1$

---

ii. $\frac{\sin 18^0}{\cos 72^0}$

$= \frac{\sin(90-72)^0}{\cos 72^0}$

$= \frac{\cos 72^0}{\cos 72^0}$

$= \mathbf{1}$

---

iii. $\text{Cos } 48^0 - \sin 42^0$

$= \text{Cos } (90\text{-}42)^0 - \sin 42^0$

$= \sin 42^0 - \sin 42^0$

$= \mathbf{0}$

---

iv. $\text{Cosec } 31^0 - \sec 59^0$

$= \sec (90\text{-}59)^0 - \sec 59^0$

$= \text{cosec } (90\text{-}59)^0 - \sec 59^0$

$= \sec 59^0 - \sec 59^0$

$= \mathbf{0}$

---

v. $\text{Cot } 34^0 - \tan 56^0$

$= \cot (90\text{-}56)^0 - \tan 59^0$

$= \tan 59^0 - \tan 59^0$

$= \mathbf{0}$

---

vi. $\frac{\sin 36^0}{\cos 54^0} - \frac{\sin 54^0}{\cos 36^0}$

$= \frac{\sin(90-54)^0}{\cos 54^0} - \frac{\sin(90-36)^0}{\cos 36^0}$

$= \frac{\cos 54^0}{\cos 54^0} - \frac{\cos 36^0}{\cos 36^0}$

$= 1 - 1$

$= \mathbf{0}$

---

vii. $\text{Sec } 70^0 \sin 20^0 - \cos 70^0 \text{ cosec } 20^0$

$= \text{Sec } (90\text{-}20)^0 \sin 20^0 - \cos 70^0 \text{ cosec}(90\text{-}70)^0$

$= \text{cosec}20^0 \sin 20^0 - \cos 70^0 \sec 70^0$

$= \frac{1}{sin20^0} \sin 20^0 - \cos 70^0 \frac{1}{cos70^0}$

= 1 - 1

= 0

viii. $\cos^2 13^0 - \sin^2 77^0$

$= \cos^2 (90-77)^0 - \sin^2 77^0$

$= \sin^2 77^0 - \sin^2 77^0$

= 0

2. Prove that:

(i) $\sin 35^0 \sin 55^0 - \cos 35^0 \cos 55^0 = 0$

LHS $= \sin 35^0 \sin (90-35)^0 - \cos 35^0 \cos(90-35)^0$

$= \sin 35^0 \cos 35^0 - \cos 35^0 \sin 35^0$

= 0 RHS

(ii) $\tan 10^0 \tan 15^0 \tan 75^0 \tan 80^0 = 1$

LHS $= \tan (90-80)^0 \tan 15^0 \tan (90-15)^0 \tan 80^0$

$= \frac{1}{tan80^0} \times \tan 80^0 \times \frac{1}{tan15^0} \times \tan 15^0$

= 1 x1

= 1 RHS

(iii) $\cos 38^0 \cos 52^0 - \sin 38^0 \sin 52^0 = 0$

LHS $= \cos (90-52)^0 \cos 52^0 - \sin 52^0 \sin(90-52)^0$

$= \sin 52^0 \cos 52^0 - \sin 52^0 \cos 52^0$

= 0 RHS

3. If $\sin 5\theta = \cos 4\theta$ where 5θ and 4θ are acute angles, find the value of θ .

$\sin 5\theta = \cos 4\theta$

$= \sin 5\theta = \cos (90-5\theta)$

$\therefore 4\theta = 90-5\theta$

$\therefore 9\theta = 90$

$\therefore \theta = \mathbf{10^0}$

4. If $\sec 4A = \operatorname{cosec} (A - 20^0)$, where $4A$ is an acute angle, find the value of 'A'

$\sec 4A = \operatorname{cosec} (A - 20^0)$

$\sec 4A = \operatorname{cosec} (90 - 4A)$

$\therefore A - 20^0 = 90 - 4A$

$\therefore$ **5**A = 70^0

$\therefore$ **A = 14^0**

EXERCISE 13.5

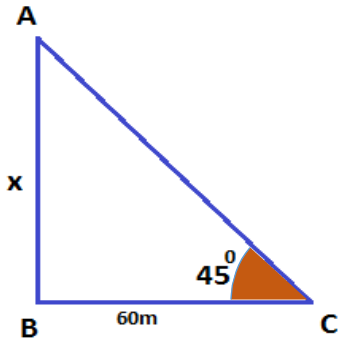
I. Find the value of 'X' .

1. $\tan\theta = \frac{AB}{BC}$

$\tan 45^0 = \frac{x}{60}$

$1 = \frac{x}{60}$

x = 60m



2. $\tan\theta = \frac{PR}{PQ}$

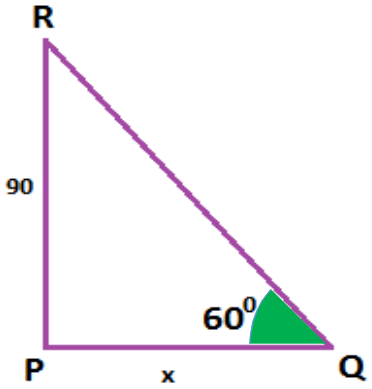
$\tan 60^0 = \frac{90}{x}$

$\sqrt{3} = \frac{90}{x}$

$x = \frac{90}{\sqrt{3}}$

$x = \frac{30\sqrt{3}.\sqrt{3}}{\sqrt{3}}$

$x = 30\sqrt{3}$

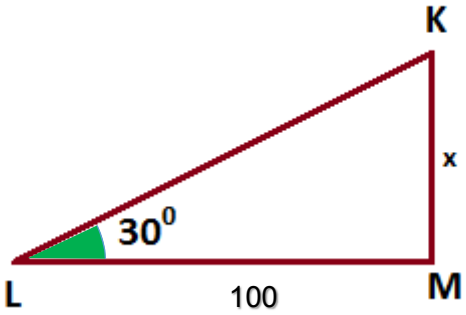


3. $\tan\theta = \frac{KM}{LM}$

$\tan 30^0 = \frac{x}{100}$

$\frac{1}{\sqrt{3}} = \frac{x}{100}$

$x = \frac{100}{\sqrt{3}}$

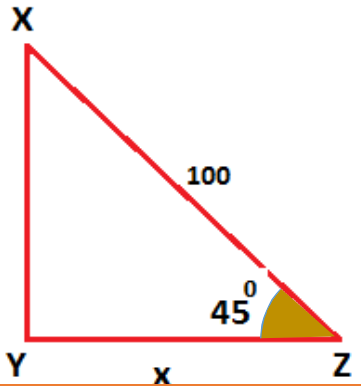


4. $\cos\theta = \frac{YZ}{XZ}$

$\cos 45^0 = \frac{x}{100}$

$\frac{1}{\sqrt{2}} = \frac{x}{100}$

$x = \frac{100}{\sqrt{2}}$



5. $\tan\theta = \frac{DF}{EF}$

$\tan x^0 = \frac{75}{75}$

$\tan x^0 = 1$

$\boldsymbol{x = 45^0}$

II. 1. A tall building casts a shadow of 300 m long when the sun's altitude (elevation)is 30°. Find the height of the tower.

Hight of the Building = AB = x ,

Length of the shadow = 300m

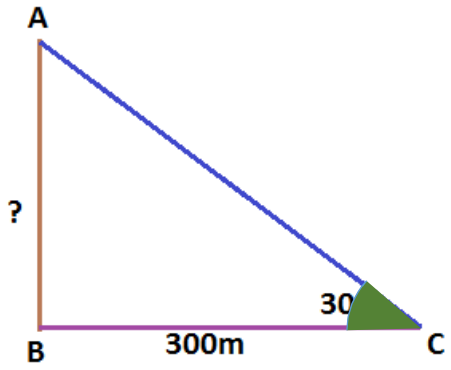
$\tan\theta = \frac{AB}{BC}$

$\tan 30^0 = \frac{x}{300}$

$\frac{1}{\sqrt{3}} = \frac{x}{300}$

$x = \frac{300}{\sqrt{3}}$

$\boldsymbol{x = 100\sqrt{3}\ m}$



1. From the top of a building $50\sqrt{3}$ m high, the angle of depression of an object on the ground is observed to be 45°. Find the distance of the object from the building..

Hight of the building = AB = $50\sqrt{3}$ m

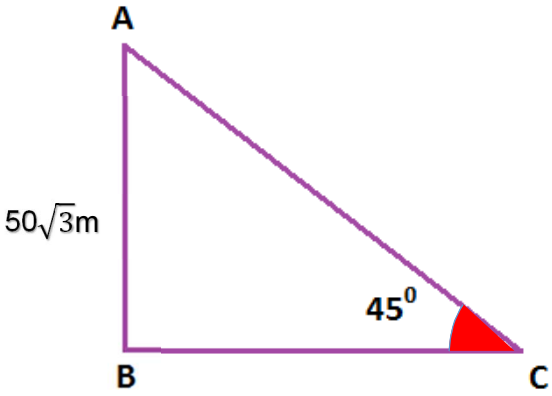
The distance from the building to the Object = BC = x

$\tan\theta = \frac{AB}{BC}$

$\tan 45^0 = \frac{x}{50\sqrt{3}}$

$1 = \frac{x}{50\sqrt{3}}$

$\boldsymbol{x = 50\sqrt{3}\ m}$



2. A tree is broken over by the wind forms a right angl ed triangle with the ground.If the broken part makes an angle of 60° with the ground and the top of the tree is now 20 m from its base, how tall was the tree?

Let the Hight of the tree = BD = (AB + AC) = (x + y) m

The height where the tree is broken = BA = x m

AC = y m

The distance from the base of the tree to the top where it is grounded = BC = 20m

$\tan\theta = \frac{AB}{BC}$

$\tan 60^0 = \frac{x}{20}$

$\sqrt{3} = \frac{x}{20}$

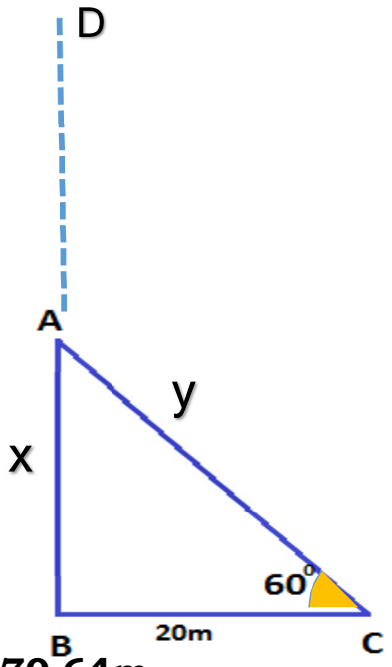
$\boldsymbol{x = 20\sqrt{3}\ m}$

$\cos\theta = \frac{BC}{AC}$

$\cos 60^0 = \frac{20}{\mathbf{20\sqrt{3}}}$

$\frac{1}{2} = \frac{20}{y}$

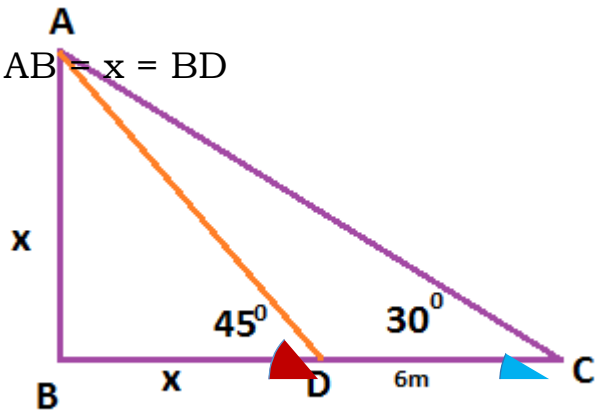
$\boldsymbol{y = 40\ m}$



∴ Hight of the tree = BD = (AB + AC)

= x + y = $20\sqrt{3}$ + 40 = 20($\sqrt{3}$ + 2) OR = 70.64m

3. The angle of elevation of the top of a flagpost from a point on a horizontal ground is found to be 30°. On walking 6 m towards the post, the elevation increased by15°. Find the height of the flagpost?



Let the height of the flagpost = AB = x = BD

$\tan\theta = \frac{AB}{BC}$

$\tan 30^0 = \frac{x}{x+6}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+6}$

$\sqrt{3}\,x = x + 6$

$\sqrt{3}\,x - x = 6$

$x(\sqrt{3} - 1) = 6$

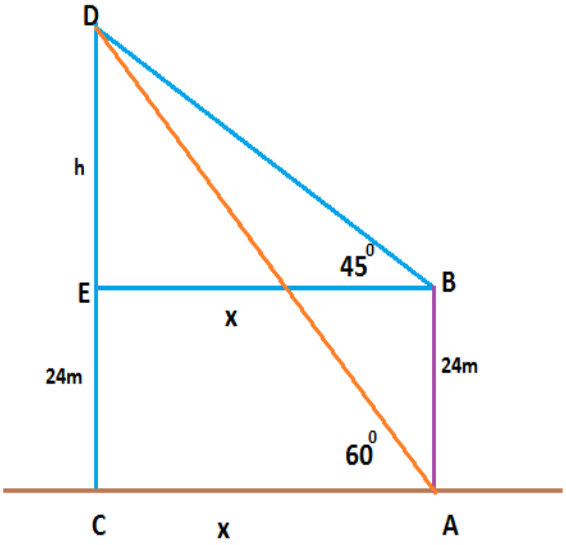
$x = \frac{6}{(\sqrt{3} - 1)}$ **m**

4. The angles of elevation of the top of a cliff as seen from the top and bottom of a building are 45° and 60° respectively. If the height of the building is 24 m, find the height of the cliff?

Hight of the cliff = CD = (24 + h)m

Hight of the building = AB = CE = 24m

AC = BE = x ಆಗಿರಲಿ



$\tan\theta = \frac{DE}{BE}$

$\tan 45^0 = \frac{h}{x}$

$1 = \frac{h}{x}$

$\Rightarrow x = h$

$\tan 60 = \frac{CD}{BE}$

$\sqrt{3} = \frac{24+h}{x}$

$\sqrt{3}\ h = 24 + h$ [$\because$ x= h]

$\sqrt{3}\ h - h = 24$

$h(\sqrt{3} - 1) = 24$

$h = \frac{24}{\sqrt{3}-1}$m

$h = \frac{24}{\sqrt{3}-1}$m

$\Rightarrow$ Hight of the cliff = $24 + h = 24 + \frac{24}{\sqrt{3}-1}$

$\Rightarrow$ Hight of the cliff = $\left(24 + \frac{24}{\sqrt{3}-1}\right)$m

---

5. From the top of a building 16 m high, the angular elevation of the top of a hill is 60° and the angular depression of the foot of the hill is 30°. Find the height of the hill.

Hight of the hill = $(h_2 + h_1)$ m

h_1 = AB = 16m ; BE = AC = x

$\tan\theta = \frac{DE}{BE}$

$\tan 30^0 = \frac{h_1}{x}$

$\frac{1}{\sqrt{3}} = \frac{16}{x}$

$\Rightarrow x = 16\sqrt{3}$ m

$\tan 60 = \frac{DE}{BE}$

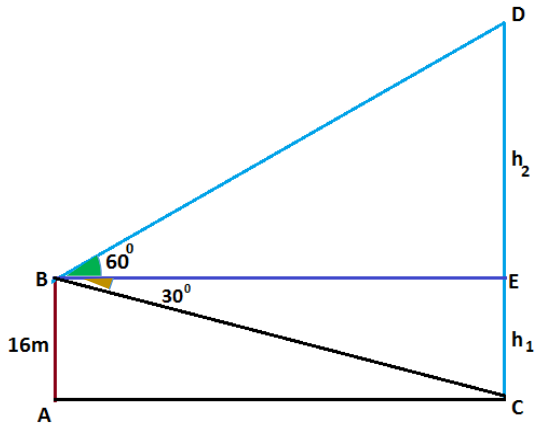
$\sqrt{3} = \frac{h_2}{16\sqrt{3}}$

$\Rightarrow h_2 = 16\sqrt{3}$ x $\sqrt{3}$

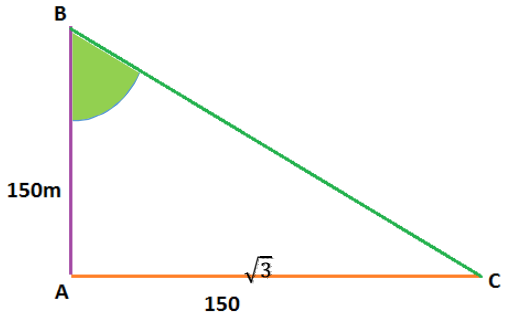
$\Rightarrow h_2 = 16$x3

$\Rightarrow h_2 = 48m$

$\therefore$ Hight of the hill = $(h_2 + h_1) = 16 + 48 = 64$m



6. Find the angle of depression if an observer 150 cm tall looks at the tip of his shadow which is $150\sqrt{3}$ cm from his foot .



$\tan\theta = \frac{AB}{AC}$

$\tan\theta^0 = \frac{150}{150\sqrt{3}}$

$\tan\theta^0 = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta^0 = 30^0$

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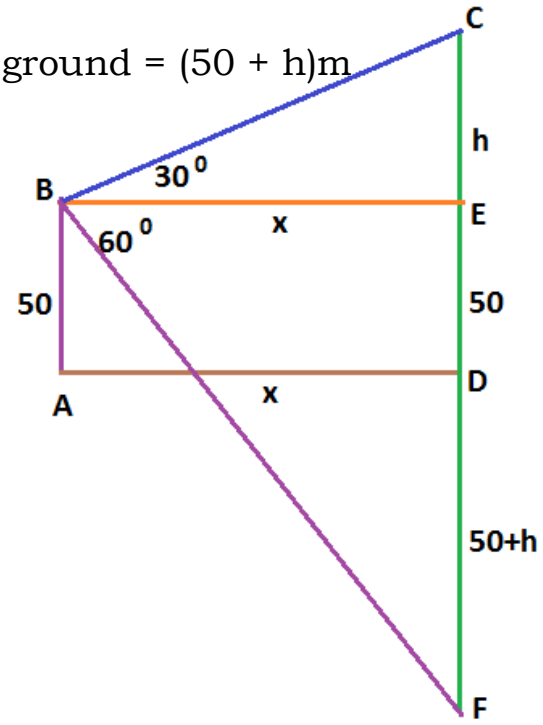
7. From a point 50 m above the ground the angle of elevation of a cloud is 30° and the angle of depression of its reflection in water is 60°. Find the height of the cloud above the ground.



The height of the cloud above the ground = (50 + h)m

$\tan 30^0 = \frac{h}{x}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$x = \sqrt{3}\,h$ ---------- (1)

$\tan 60^0 = \frac{50+50+h}{x}$

$\sqrt{3} = \frac{50+50+h}{x}$

$x = \frac{100+h}{\sqrt{3}}$ ----------(2)

$\sqrt{3}\,h = \frac{100+h}{\sqrt{3}}$ [From(1) and (2)]

$3\,h = 100 + h$

$\Rightarrow 2h = 100$

$\Rightarrow h = 50m$

$\Rightarrow$The Require height = (50 + h) = 50 + 50 = 100m